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LETTER TO THE EDITOR

Anisotropy of the London penetration depth in layered superconductors

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Abstract. We find the temperature dependence of the anisotropy of the London penetration depth λ in a layered superconductor. The system considered is an array of sheets coupled by the Josephson interaction. We consider separately the cases when the sheets are identical, when they have varying order parameters and finally when every other sheet becomes normal individually. We show that the anisotropy of λ is temperature independent only if all the layers are identical.

The new cuprate superconductors are layered systems that show a high degree of anisotropy in their electronic structure. In the Bi– and Tl–Cu compounds, the ratio of effective masses parallel to and perpendicular to the layers can be as high as several thousands [1–4]. An important tool that can be used to measure this anisotropy is the London penetration depth λ. The value of λ is theoretically not difficult to calculate and a variety of methods exist for measuring it (measurement of diamagnetism of small particles of different sizes [5–7] or of small single crystals [8, 9], μSR [10–12], lower critical field H_{c1} [13] and high-resolution torque magnetometry [4, 14]).

In this work we calculate the dependence of the anisotropy $A(T) = \lambda_z^2(T)/\lambda_\parallel^2(T)$ as a function of temperature T (λ_z and $\lambda_\parallel$ are the London penetration depths perpendicular and parallel, respectively, to the layers). We establish a model of a system composed of an array of two-dimensional layers with the tight-binding approximation to treat the electron motion between the layers. In our model the pairing interaction may be different inside the different layers. We calculate the function $A(T)$ using such a model. This in turn allows us to make measurements of $A(T)$ in real compounds and to make predictions about the electronic structure inside these compounds.

In the model under study the anisotropy parameter A depends strongly on the value of the transfer integral t which characterizes the band width in the c direction (perpendicular to the layers). If the anisotropy is not very large, i.e. the conditions $T_c \ll t \ll \varepsilon_F$ or equivalently $d \ll \xi_z \ll \xi_\parallel$ are fulfilled, all the individual superconducting parameters of the layers are effectively averaged owing to the strong proximity effect. Here ε_F is the Fermi energy inside the layers given by $\hbar^2 nd/m_\parallel$, n is the three-dimensional concentration of conducting electrons, d is the interlayer distance, $m_\parallel$ is the effective mass of electrons along the layers, and $\xi_\parallel$ and ξ_z are coherence lengths along layers and in the perpendicular direction (along the z axis). In this case we obtain the usual 3D anisotropic superconductor with the value A which is independent of T and of the strength of the pairing interactions inside the layers.

In the following we consider the more interesting case of the Josephson coupling of layers, i.e. the system with $t \ll T_c$ (or $\xi_z \ll d$) will be under study [15]. In this limiting case, A does depend on T if the system consists of layers with different intralayer pairing interaction.

Josephson coupling seems to model the Bi and Tl compounds properly, as evidenced by their high degree of anisotropy. The superconducting coherence length $\xi_{\parallel}(0)$ in CuO_2 layers is about 15–20 Å [1]. As anisotropy is about several thousands in these systems, this gives $\xi_z(0) \simeq 0.2$ Å, which is much smaller than the typical interlayer separation d of the order of 10 Å. The fact that in these compounds the inequality $\xi_z(0) \ll d$ or $t \ll T_c$, the criterion for Josephson coupling between the layers [15], is satisfied indicates that our model may be applicable to these systems.

We consider a simple model of an array of sheets with alternating coupling constant Λ_1 and Λ_2 in the sheets (i.e. bare critical temperatures T_{c1} and T_{c2} are defined on the individual layers). We choose $T_{c1} > T_{c2}$. We describe the transfer of electrons between layers in the framework of the tight-binding approximation. Thus we write the wavefunctions of electrons in the Wannier representation along the c axis and in the Bloch representation inside the sheets. We introduce alternating transfer integrals t and t' between the neighbouring sheets. We find it useful to define the nth unit cell as consisting of layers 1 and 2 with coupling parameters Λ_1 and Λ_2, and transfer integral t between the sheets inside the unit cell.

Identical sheets ($\Lambda_1 = \Lambda_2$) with $t = t' \ll T_c$ are described by the Lowrence–Doniach model, i.e. by a model in which there is Josephson coupling between identical sheets. In general, the case $\Lambda_1 \neq \Lambda_2$ and $t, t' \ll T_{c1}$, called the S–S' system, is described by Josephson coupling of different layers, with the S–N system (with $\Lambda_2 = 0$) being the limiting case. If $t \gg T_{c1}$ but $t' \ll T_{c1}$, we have the effective averaging of coupling inside the cell and the system is similar to an array of identical sheets S–S with one transfer integral t'. In this case, the sheet S consists of two layers with an effective intrasheet coupling constant $(\Lambda_1 + \Lambda_2)/2$. In the following we consider two cases: a Josephson S–S system ($\Lambda_1 = \Lambda_2$ and $t = t' \ll T_{c1}$) and a Josephson S–S' or S–N system ($\Lambda_1 \neq \Lambda_2$ and $t, t' \ll T_{c1}$) on the assumption that all other parameters of layers 1 and 2 are identical. The generalization of our results to the case of different parameters of layers 1 and 2 in the normal phase is straightforward.

The Josephson S–S or S–S' model may be applied to Bi and Tl compounds. We label the CuO_2 sheets with index 1 and the BiO or TiO layers with index 2 assuming that they are metallic (if they are insulating we get the system S–S). The S–S, S–S' or S–N models could be applied also to artificially fabricated crystals with alternating layers of $YBa_2Cu_3O_7$ and $PrBa_2Cu_3O_7$ [16, 17]. The latter alone is a magnetic insulator but, when combined with $YBa_2Cu_3O_7$, its properties are unknown. In terms of our model, this system would be characterized as either S–S or S–N with the $PrBa_2Cu_3O_7$ layer acting as an insulator or metal, respectively.

We shall show that the temperature dependence of the anisotopy $A(T)$ is quite different in the models S–S, S–S' and S–N. So measurement of the anisotropy would indicate which model is preferred.

We assume the usual BCS isotropic behaviour within the layer and use the following Hamiltonian:

$$H = H_0 + H_t$$

$$H_0 = \sum_{pn\alpha\sigma} [\varepsilon(p) - \varepsilon_F] a_{p,n,\alpha,\sigma}^+ a_{p,n,\alpha,\sigma} + \tfrac{1}{2} \sum_{pp'n\alpha} \Lambda_\alpha a_{p,n,\alpha,\sigma}^+ a_{-p,n,\alpha,-\sigma}^+ a_{-p',n,\alpha,-\sigma} a_{p',n,\alpha,\sigma}$$

$$H_t = \sum_{pp'n\alpha\sigma} f_{pp'} (t a_{p,n,1,\sigma}^+ a_{p,n,2,\sigma} + t' a_{p,n+1,2,\sigma}^+ a_{p',n,1,\sigma}) \tag{1}$$

$$+ f^*_{-p,-p'} (t a^+_{p,n,1,\sigma} a_{p',n+1,2,\sigma} + t a^+_{p,n,2,\sigma} a_{p',n,1,\sigma}) \qquad \varepsilon(p) = p^2/2m_\parallel$$

$$f_{pp'} = \int dr \exp[i\chi(r) + i(p - p') \cdot r] \qquad \chi(r) = (ed/\hbar c) A_z(r)$$

where α labels the layer inside the unit cell, σ is the spin index, n labels the unit cell, $r = (x, y)$ and p and p' are two-dimensional momenta inside the layer. The parallel component of the vector potential can be taken into account in the usual way by changing p to $p - eA_\parallel/\hbar c$ in $\varepsilon(p)$.

The current $j_\parallel$ that flows along the layers in response to $A_\parallel$ is given by the standard expression [18] neglecting the terms which are of second order in t/T_{c1}:

$$j_\parallel(k_\parallel) = \frac{e^2}{cd} A_\parallel(k_\parallel) N(0) v_F^2 T \sum_{\omega\alpha} \Delta_\alpha^2 (\omega^2 + \Delta_\alpha^2)^{-3/2} \qquad (2)$$

where $j_\parallel(k_\parallel)$ and $A_\parallel(k_\parallel)$ are the Fourier components of the current and vector potential in k-space, respectively, $N(0) = m_\parallel/2\pi\hbar^2$ is the density of states at the 2D Fermi surface, v_F is the 2D Fermi velocity and Δ_α is the order parameter on the layer α. This expression is valid in the limit $k_\parallel \ll T_{c1}/v_F$, i.e. $\lambda_\parallel \gg \xi_\parallel$, which is true of high-$T_c$ superconductors. From (2) we get $\lambda_\parallel(T)$.

To obtain the perpendicular current we calculate the addition to the free energy F_a arising from the vector potential A_z to second order in A_z through perturbation theory. The current j_z is given by $j_z = c \, \delta F_a/\delta A_z$. The relationship between j_z and A_z gives the penetration depth λ_z.

If Δ_1 and Δ_2 are large (as is true over most of the temperature range $T < T_{c2}$ in the cases S–S and S–S'), we may consider only the local effect of neighbouring layers on the free energy. So at large Δ_2 we consider a simplified system with only two layers with coupling constants Λ_1, Λ_2 and between which are the transfer integrals t and t' alternately. Using the perturbation theory in t and t' we obtain

$$F_a = T \sum_{n\omega} \int dp \, |\Delta_1 \Delta_2| \frac{[t^2 \cos(\varphi_{n1} - \varphi_{n2}) + t'^2 \cos(\varphi_{n+1,2} - \varphi_{n1})]}{(\omega^2 + \xi^2 + |\Delta_1^2|)(\omega^2 + \xi^2 + |\Delta_2^2|)} \qquad (3)$$

$$\varphi_{n1} - \varphi_{n2} = (2ed/\hbar c) A_z \qquad \xi = \varepsilon(p) - \varepsilon_F$$

where $\varphi_{n\alpha}$ is the phase of the order parameter on the layer $n\alpha$. Only the anomalous Gor'kov functions $F_{n\alpha}(\omega, p)$ were used to obtain equation (3) because they contain the phase difference between the order parameters, i.e. $\cos(\varphi_{n1} - \varphi_{n2})$, which is then expanded to second order in A_z.

In the S–S system, i.e. if $T_{c1}/T_{c2} - 1 \ll (t^2 + t'^2)/T_{c1}^2$, we obtain for the anisotropy parameter

$$A = \hbar^2 v_F^2/2d^2[\min(t^2, t')] \qquad (4)$$

on the assumption that d is much smaller than the length over which A_z changes. This parameter is simply the ratio of effective masses $m_z/m_\parallel$. The value m_z is given by the expressions

$$m_z^{-1} = \langle v_z^2 \rangle/2\varepsilon_F \qquad v_z = (1/\hbar d)(\partial \varepsilon/\partial q)$$

$$\varepsilon(p, q) = p^2/2m_\parallel + (t^2 + t'^2 + 2tt' \cos q)^{1/2} \qquad (5)$$

where $\varepsilon(p, q)$ is the electron spectrum which corresponds to the Hamiltonian (1); the angular brackets indicate averaging of the electron velocity over the Fermi surface including the averaging over q in the interval $0 \leq q \leq 2\pi$. We note that under conditions $t, t' \ll \varepsilon_F$ we get the open Fermi surface and then $m_z^{-1} = [\min(t^2, t'^2)]d^2/\varepsilon_F\hbar^2$.

According to (4) the anisotropy does not depend on T for the S–S system, because the identical sums over Matsubara frequencies appear in both $j_{\parallel}$ and j_z and cancel each other in A.

In S–S' system, i.e. for $\ln(T_{c1}/T_{c2}) \gg (t^2 + t'^2)/T_{c1}^2$, we obtain A, multiplying the right-hand side of (4) by a factor Δ_1/Δ_2, which gives the anisotropy a temperature dependence. Near T_{c1}, this ratio is determined by the Ginzburg–Landau functional for two layers (which is itself determined by the equations of self-consistency for the order parameters Δ_α):

$$a_1(T)\Delta_1 + \beta\Delta_1^3 + \eta(\Delta_2 - \Delta_1) = 0$$
$$a_2(T)\Delta_2 + \eta(\Delta_1 - \Delta_2) = 0 \tag{6}$$

where we have linearized the equation for the order parameter Δ_2, which is much smaller than Δ_1 near T_{c1}. In (6) we get $a_\alpha(T) = \ln(T_{c\alpha}/T)$, $\beta = -K/T^2$, $\eta = K(t^2 + t'^2)/2T^2$ and $K = 7\zeta(3)/8 \simeq 0.1$. This yields $\Delta_1/\Delta_2 = -[2T^2 \ln(T_{c2}/T)]/K(t^2 + t'^2)$ near T_{c1} and well above T_{c2}. At $T \ll T_{c2}$ we get $\Delta_1/\Delta_2 = T_{c1}/T_{c2}$, which is assumed to be much less than Δ_1/Δ_2 at $T \simeq T_{c1}$. We see therefore that, if the coupling constants differ greatly, anisotropy increases with increasing temperature.

In the case of the S–N system, we must know the full Green functions for a system with the Hamiltonian (1). We introduce the Green functions $G_{\alpha,\beta}(\omega, p, n, m)$ and $F_{\alpha,\beta}^+(\omega, p, n, m)$ in the usual way (see [18]) and obtain the matrix equation

$$
\begin{bmatrix}
\omega_+ & -T(q) & \Delta_1 & 0 \\
-T^*(q) & \omega_+ & 0 & \Delta_2 \\
\Delta_1 & 0 & \omega_- & T(q) \\
0 & \Delta_2 & T^*(q) & \omega_-
\end{bmatrix}
\begin{bmatrix}
G_{1\alpha} \\
G_{2\alpha} \\
F_{1\alpha}^+ \\
F_{2\alpha}^+
\end{bmatrix}
=
\begin{bmatrix}
\delta_{1\alpha} \\
\delta_{2\alpha} \\
0 \\
0
\end{bmatrix}
\tag{7}
$$

for the Fourier components of discrete variable $n - m$, $\omega_{\pm} = i\omega \pm \xi$ and $T(q) = t + t' \exp(i\,q)$. The order parameters Δ_1 and Δ_2 are given by the usual self-consistency equations

$$\Delta_\alpha = \frac{\Lambda_\alpha T}{(2\pi)^3} \sum_\omega \mathrm{d}p\,\mathrm{d}q\, F_{\alpha,\alpha}^+(\omega, p, q) \tag{8}$$

and, in the S–N system, $\Delta_2 = 0$ because $\Lambda_2 = 0$. The solution of equation (7) with accuracy $(t/T_{c1})^2$ can be written as

$$G_{11}(\omega, p, q) = \omega_-[\omega_+^2 - \Delta_1 d(q)]D^{-1}$$
$$G_{22}(\omega, p, q) = [\omega_+^2 \omega_- - \Delta_1^2 \omega_+ - \Delta_1 \omega_- d(q)]D^{-1}$$
$$G_{12}(\omega, p, q) = G_{21}(\omega, p, -q) = T(q)[\omega_+^2 - \Delta_1 d(q)]D^{-1}$$
$$F_{11}^+(\omega, p, q) = -\Delta_1 \omega_+ \omega_- D^{-1}$$
$$F_{22}^+(\omega, p, q) = \Delta_1^2 d(q)D^{-1} \tag{9}$$
$$F_{12}^+(\omega, p, q) = F_{12}(-\omega, p, q) = T(q)\Delta_1 \omega_+ D^{-1}$$
$$F_{21}^+(\omega, p, q) = F_{21}(-\omega, p, q) = T^*(q)\Delta_1 \omega_- D^{-1}$$
$$D = (\omega^2 + \mathrm{d}\xi^2 + \Delta_1^2)[\omega^2 + \xi^2 + d^2(q)]$$
$$d(q) = |T(q)|^2/\Delta_1 .$$

We note that at low temperatures and low energies ξ the function $F_{22}^+(\omega, p, q)$ of layers N has the usual form with effective gap $d(q) = (t^2 + t'^2 + 2tt' \cos q)/\Delta_1$ while we get the gap Δ_1 on layers S. The gap $d(q)$ characterizes the induced superconductivity of layers N owing to the proximity effect. Such a gap of layers N forms gradually below the temperature $T_{\text{eff}} \simeq (t^2 + t'^2)/\Delta_1$ which is the effective critical temperature of layers N (there is no real phase transition at T_{eff}; for more details see [19]).

Using the Green functions (9) we can calculate the parallel and perpendicular currents by the perturbation theory in vector potentials $A_\parallel$ and A_z for the free energy. Calculating the contribution to the free energy from $A_\parallel$ we can omit the Green functions which are non-diagonal in α. The result at $T \leqslant T_{\text{eff}}$ is

$$\lambda_\parallel^{-2} = \frac{4\pi e^2 N(0) v_F^2}{c^2 d} \left(1 + \frac{T}{\pi} \int_0^\pi dq \sum_\omega \frac{d^2(q)}{[\omega^2 + d^2(q)]^{3/2}} \right)$$

$$\lambda_z^{-2} = \frac{8 e^2 t^2 N(0) T d}{\hbar^2 c^2} \sum_\omega \int d\xi \int_0^\pi dq \frac{\Delta_1 d(q)}{(\omega^2 + \xi^2 + \Delta_1^2)[\omega^2 + \xi^2 + d^2(q)]} \tag{10}$$

$$A(T) = (\hbar^2 v_F^2 \Delta_1^2 / 64\pi t^4 d^2)[\ln(\Delta_1/T_{\text{eff}})]^{-1} \qquad T \leqslant T_{\text{eff}}.$$

We note that layers N give about the same contribution to $j_\parallel$ as the layers S below T_{eff}, i.e. they are completely superconducting at low temperatures.

At $T_{\text{eff}} \ll T \ll T_{c1}$ we get only the contribution of layers 1 to $j_\parallel$ (the first term in square brackets in (9)) because the induced superconductivity of layers N is negligible there. Using (9) we get with logarithmic accuracy

$$A(T) = 0.02(\hbar^2 v_F^2 T_{c1}^2 / t^4 d^2)[\ln(\Delta_1/T)]^{-1} \qquad T_{\text{eff}} \ll T \ll T_{c1}. \tag{11}$$

Near T_c the expression for $A(T)$ is

$$A(T) = 7\pi^2 \zeta(3)\hbar^2 v_F^2 T_{c1}^2 / 186 t^4 d^2 \zeta(5) = 0.44 \hbar^2 v_F^2 T_{c1}^2 / t^4 d^2. \tag{12}$$

Comparison of (10)–(12) shows that the magnetic anisotropy in the S–N system increases with increasing temperature.

We have shown that the magnetic anisotropy parameter A is temperature independent in the Josephson S–S system while it increases markedly with increasing temperature in the S–S′ and S–N systems.

There is some hope that these results can be applied to Bi and Tl compounds and to artificial superlattices of $YBa_2Cu_3O_7/PrBa_2Cu_3O_7$ type. Inasmuch as only one of the transfer integrals in the Bi and Tl compounds is known to be much less than T_c from the value A, we do not know whether we may model these systems as S–S ($t' \ll T_c$, $t \gg T_c$) or S–S′ ($t, t' \ll T_c$). For $Tl_2Sr_2CaCu_2O_8$ with $T_c = 100$ K from the torque measurements the value $A \simeq 10^4$ was obtained in the temperature interval 90–100 K [4]. The fact that A is independent of T in this interval means that all layers coupled by the Josephson interaction are identical with respect to the individual critical temperature. We can conclude then that, if TlO layers are conducting, they are joined to the CuO_2 layers by a strong intracell transfer integral $t \gg t_c$ to form a sheet and the properties of the individual layers are averaged over such a sheet. Then the sheets from different unit cells are coupled by the Josephson interaction and the small transfer integral $t' \ll T_c$ is really an intercell transfer integral. Alternatively, the TlO layers can be insulating.

Similar measurements of the magnetic anisotropy of the system $YBa_2Cu_3O_7/PrBa_2Cu_3O_7$ might indicate the preferred model here as well: Josephson S–N (with the Pr layers acting as a metal) or S–S (with the Pr layer acting as an insulator or as a metal but in the case of strong proximity effect, i.e. $t \gg T_c$).

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